U.G. 6th Semester Examination - 2022

MATHEMATICS

[PROGRAM]

Course Code : BMTMDSRT-3 & 4 (DSE 3 & 4)

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

This question papers contains both DSE 3 & 4. Students are thereby instructed to answer DSE paper out of these two (DSE 3 & DSE 4) as he/she opted for.

Title : Probability and Statistics

Code : BMTMDSRT3 (DSE 3)

1. Answer any **ten** questions: $1\times10=10$

a) Show that for any two events A and B,

$$P(\bar{A}\cup B)=1-P(A)+P(A\cap B)$$

where $\bar{A}$ is the complement of A.

b) If A and B are two independent events, then show that $\bar{A}$ and $\bar{B}$ are also independent.

c) Find a value of the constant k such that

$$f(x)=kx(1-x),\ 0<x<1$$
$$=0,\ \text{elsewhere;}$$

is a probability density function.

d) For any two events A and B with P(A)>0, show that $P(B/A)\geq 1-\dfrac{P(B)}{P(A)}$.

e) Fill in the blank : If E(X) =m and a, b are constants then E(aX+b) = __________.

f) Write down the probability mass function of Poisson μ-variate.

g) Given that 'X' has a Poisson distribution with Variance 0.5. Calculate P(X=3).

h) Let X be a random variable having the following distribution:

$$P(X=-1)=\frac{1}{3},\ P(X=0)=\frac{1}{2},\ P(X=1)=\frac{1}{6}$$

Find $E(X^2)$.

i) State Bayes' theorem.

j) Write down the probability density function of the standard normal distribution.

k) Find the first two moments about 5 of the following data

6, 7, 10, 9, 11

l) What do you mean by a 'null hypothesis'?

m) Show that the sample mean is unbiased estimator of population mean.

n) If $u=2x$ and $v=3y$ then show that the correlation co-efficient between u and v is equal to the correlation co-efficient between x and y.

o) If the mode of a variable X is 6.7, what would be the mode of $Y = 3X + 2$?

2. Answer any **five** questions: $2\times5=10$

a) Let X be a binomial (n, p) variate. Find the moment generating function of X.

b) If X be a random variable such that $E(X^2)$ exists, then show that $Var(X)=E(X^2)-\{E(X)\}^2$.

c) Let X be a random variable with the following probability distribution:

x	–3	6	9
P(X=x)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Evaluate $E\{(2X+1)^2\}$

d) The density function of a two-dimensional random variable (X, Y) is given by

$$f(x,y)=\begin{cases}3xy(x+y), 0\le x\le 1,\ 0\le y\le 1\\ 0, \text{ otherwise}\end{cases}$$

Find the marginal density function of X and Y.

e) If $\{T_n\}$ is a sequence of estimator such that $E(T_n)\to\theta$ and $Var(T_n)\to 0$ as $n\to\infty$, then show that $\{T_n\}$ is a consistent estimator for θ.

f) Draw the scatter diagram when the correlation co-efficient is +1, –1.

g) If 'X' be normal (0, 1) distribution, find the distribution of e^{-X}.

h) Write down the maximum likelihood functions for the normal (m,σ) population.

3. Answer any **two** questions: $5\times2=10$

a) Show that Poisson distribution is the limiting case of Binomial distribution under certain condition to be stated by you.

b) If the joint probability density function of a two-dimensional random variable (X, Y) is given by

$$f(x,y)=\frac{1}{28}(3x+y),\ \ 1\le x\le 3;\ \ 0\le y\le 2$$
$$=0,\quad \text{elsewhere}$$

then find the marginal distribution of X and Y. Investigate whether X and Y are independent.

c) Calculate the correlation co-efficient and determine the regression lines of Y on X and X on Y for the following sample:

X	8	10	5	8	9
Y	1	3	1	2	3

4. Answer any **one** question: 10×1=10

a) i) The probability density function of a continuous random variable X is given by

$$f(x)=\begin{cases}Ke^{-b(x-a)}, & \text{if } a\le x<\infty\\ 0, & \text{elsewhere.}\end{cases}$$

If m, σ are the mean and standard deviation of X, then show that $K=b=\sigma^{-1}$ and $a=m-\sigma$.

ii) The joint probability density function of the random variables X and Y is

$$f(x,y)=\begin{cases}8xy, & 0\le x\le 1,\ \ 0\le y\le 1\\ 0, & \text{elsewhere.}\end{cases}$$

Examine whether X and Y are independent. 6+4=10

b) i) Define distribution function of a continuous random variable X having probability density function f(x). If f(x) be the distribution function of X, then show that

$$P(a\le X\le b)=F(b)-F(a).$$

ii) Let the two dimensional continuous random variable (X, Y) has the joint p.d.f. given by

$$f(x,y)=\begin{cases}6x^2y, & 0<x<1,\ 0<y<1\\ 0, & \text{elsewhere.}\end{cases}$$

Find $P\left(0<X<\frac{3}{4},\frac{1}{3}<Y<2\right)$.

iii) Let X_1 and X_2 be two independent random variables with means 5 and 10 and standard deviations 9 and 12 respectively. Obtain ρ (U, V), where $U=3X_1+4X_2$ and $V=3X_1-X_2$. (1+2)+3+4=10

c) i) Consider a random sample of size n without replacement from a finite population of size N and variance σ^2. Show that the variance of the sample mean is $\frac{\sigma^2}{n}.\frac{N-n}{N-1}$.

ii) If Z has a standard normal distribution and U has a chi-square distribution with n-degrees of freedom and if Z and U are independent, then show that

$$t=\frac{Z}{\sqrt{U/n}}$$

has a t-distribution with n-degrees of freedom. 5+5

Title : Mechanics-II

Code : BMTMDSRT4 (DSE 4)

1. Answer any **ten** questions: 1×10=10

a) Define coplanar forces.

b) What do you mean by virtual work?

c) Define the moment of a force.

d) State the conditions that a given system of forces may reduce to a single resultant force acting on a rigid body.

e) Define pressure of a fluid at a given point.

f) Write down stress matrix of a moving perfect fluid.

g) What will be the dimension of

i) Shearing stress

ii) Pressure

h) Define ideal fluid.

i) Define compressible fluid.

j) Mention a force which will not appear in the equation of virtual work.

k) What is the equi-density surface of a fluid?

l) Define body force with an example.

m) State Boyle's law.

n) Can a fluid be simultaneously non-homogeneous and incompressible?

o) What is wrench of a system of forces acting on a body?

2. Answer any **five** questions: $2\times5=10$

a) State the converse of the principle of virtual work.

b) What do you understand by an effective surface and a free surface of a fluid?

c) Define the centre of pressure of a plane lamina.

d) Write down the conditions of equilibrium of a freely floating body in a liquid.

e) Define stable and unstable equilibrium of a body with examples.

f) Define a continuous body. When is it called deformable?

g) What do you understand by Isothermal and Adiabatic changes of a gas?

h) Define common catenary and write down the cartesian equation of a common catenary.

3. Answer any **two** questions: $5\times2=10$

a) A rhombus ABCD is formed of four equal uniform rods freely jointed together and suspended from the point A. It is kept in position by a light rod joining the mid-points of BC and CD. If T be the thrust in this rod and W be the weight of the rhombus then show that $T = W\tan\frac{A}{2}$. 5

b) i) Show that a force and a couple cannot produce equilbrium.

ii) State the necessary and sufficient condition for equilbrium of a fluid under the action of external forces. 3+2

c) Show that the components of forces, respresented by $X=\mu y(y+z)$, $Y=\mu z(z+x)$ and $Z=\mu y(y-x)$ will keep a mass of fluid at rest. Find the equations of equi-pressure surfaces. 5

4. Answer any **one** question: $10\times1=10$

a) i) Derive the pressure equation of a mass of fluid in equilibrium under a given system of external forces.

ii) A liquid fills the lower half of a circular tube of radius 'a' in a vertical plane. If the tube is now rotated about the vertical diameter with uniform angular velocity ω such that the liquid is about to separate in two parts, then show that $\omega = \sqrt{2g/a}$.

5+5

b) i) Two forces one of which acts along Z-axis, are together equivalent to force components (X, Y, Z) and couple components (L, M, N) referred to a rectangular cartesian axes. Show that the magnitude of the force along Z-axis is (LX+MY+NZ)/N.

ii) Discuss the stability of a system having one degree of freedom. 6+4

c) i) State and prove the pressure-volume relation for a perfect gas in adiabatic change of state.

ii) A cone whose vertical angle is 2α, has its lowest generator horizontal and is filled with liquid. Prove that the resultant thrust on the curved surface is $\sqrt{1+15\sin^2\alpha}$ times the weight of the liquid. 4+6

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